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Magnetoconvective heat transfer from a cylinder under the influence of a nonuniform magnetic field

E Blums, A Mezulis and G Kronkalns

Institute of Physics, Latvian University, Miera 32, Salaspils LV-2169, Latvia

E-mail: eblums@sal.lv

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Abstract
In this paper, we present experimental results on heat transfer from a nonmagnetic cylinder to a magnetic fluid under the influence of transverse laminar free convection. Measurements are performed in the presence of uniform and nonuniform external magnetic fields; both the fields and their main gradients are directed transversely to the cylinder axis. In zero field the measurement results for both the unsteady and the stationary heat transfer ($Ra > 10^6$) agree well with the dependences found in the framework of a boundary layer approximation. If a nonuniform magnetic field is applied, the theoretically predicted additive action of gravitation and magnetic convection on the heat transfer intensity is confirmed. In the presence of a uniform field, the changes of heat transfer intensity are insignificant despite the action of strong internal field gradients induced by the nonmagnetic cylinders themselves.

1. Introduction

The augmentation of heat transfer intensity is an important problem not only in power engineering but also in many other disciplines of modern technology, including the cooling of heat generating elements in microelectronics. The heat dissipation density of compact electronics exceeds several hundreds of W cm^{-2}. It is obvious that removal of such heat significantly exceeds the limit of conventional air coolers. Liquid cooling devices with microchannel heat sinks provide the possibility of removing heat fluxes strongly beyond the air cooling limit. However, providing the necessary liquid flow rate requires much energy and the cooler is technically complicated. Recently, an idea for developing liquid metal coolers equipped with MHD pumps appeared [1]. Unfortunately, such devices have several disadvantages (chemical activity of low temperature liquid metals and alloys, necessity to employ low voltage energy supplies for conductive MHD pumps). The objective of the present work is to estimate the thermomagnetic convection in ferrofluids as an alternative to the magnetohydrodynamic cooling systems. Of course, the ferrofluids cannot compete with liquid metals as cooling agents due to the relatively low thermal conductivity, but they have a significant technical advantage because of the possibility of initiating a convective motion and a heat transfer by the heat sources themselves without the necessity to employ any external energy resources. Many authors, starting from the early 1970s, have investigated the problem of thermomagnetic convection. The research interests were focused mainly on the basic problems of thermoconvective instabilities under the influence of a uniform external magnetic field (a review of the main results is given in [2] and [3]). Only a few works are devoted to the heat transfer in the presence of a non-homogeneous field. First experiments with paramagnetic electrolyte solutions (1972) and further more detailed investigations of convection in ferrofluids confirm the general similarity between the gravitation and the magnetic convection. Several qualitative heat transfer measurements indicate the effect of magnetoviscosity and of local magnetic field gradients on heat transfer from nonmagnetic and ferromagnetic bodies [4]. In this paper we focus our attention on thermomagnetoconvective augmentation of heat transfer from a cylindrical body to the surrounding liquid by applying a non-homogeneous magnetic field and by using specially designed ferrocolloids of high pyromagnetic coefficients.

2. Theoretical background

If the ferrofluid is exposed to a nonuniform magnetic field, besides the conventional gravitation force on the fluid there acts an additional magnetostatic body force. Under the approximation of equilibrium magnetization $\mathbf{M} = \chi\mathbf{H}$ with magnetic susceptibility χ being dependent solely upon the field and the density $\rho = \text{const}$, not only the gravitation force but also the magnetic one is a potential, and the convection inside the fluid cannot arise. The non-potentiality of bulk forces appears only if a fluid possesses the spatial nonuniformity of ρ and M due to their dependence on temperature. The condition of appearance of the non-threshold convection is the following:

$$\text{rot}\,\mathbf{f} = \nabla T \times \left[\beta_T \rho_0 \mathbf{g} \pm \mu_0 \beta_m \frac{\chi_0(H)}{2} \nabla H^2\right] \neq 0 \quad (1)$$

where the sign $\pm$ means parallel or anti-parallel orientation of the magnetic field gradient with respect to the gravitation force, β_T and β_m are the relative volumetric expansion and relative pyromagnetic coefficients. It is obvious that a deep similarity between the thermogravitational and the thermomagnetic convection exists. The heat transfer intensity is determined by a single Rayleigh number Ra, consisting of two (gravitation and magnetic) parts:

$$Ra = Ra_\text{T} + Tm_\text{T} = \frac{\rho c_\text{T} d^3 \Delta T}{\eta\lambda} (\beta_T \rho g \pm \mu_0 \beta_m M \nabla H) \quad (2)$$

(the symbols here are those conventionally used for viscosity, thermal conductivity, heat capacity, temperature difference and length scale). Numerical evaluation says that even in small laboratory experiments both the thermal (Ra_T) and the magnetic (Ra_m) Rayleigh numbers are relatively high; they can exceed the values 10^5–10^6. This means that the problem of convective heat transfer may be theoretically considered in the framework of a boundary layer approximation.

The velocity inside the boundary layer develops under the action of a longitudinal component of the body force $f_x = \alpha(x)$; x is the running coordinate along the boundary surface. The convection near flat and axisymmetric bodies is considered in [5]. Employing the concept of local similarity and introducing a polynomial approximation of velocity and temperature profiles, the analysis of integral boundary layer equations shows that the heat transfer monotonically builds up with growing Prandtl numbers $Pr = \eta c_\text{T}/\lambda$. In liquids with $Pr \approx 10$ (typical values for magnetic fluids), the heat transfer saturates. The local Nusselt number $Nu = q_w d(\lambda\Delta T)^{-1}$ (q_w is the local heat flux on the body surface) follows a simple formula found for asymptotic values of the Prandtl number $Pr \to \infty$:

$$\frac{Nu}{\sqrt[4]{Ra}} = 0.502\,75(r(x)\alpha(x))^{1/3} \times \left[\int_0^x (r(x)^4\alpha(x))^{1/3}\,\text{d}x\right]^{-1/4}. \quad (3)$$

For pure gravitation 2D convection ($\mathbf{g} = \text{const}$) near horizontal cylinders the functions in the equation (3) are the following: $r(x) = 1$ and $\alpha(x) = \sin(x)$. For thermomagnetic convection such simple dependence is not valid because Maxwell equations do not allow spatial uniformity of the magnetic acceleration, $\nabla H^2 \neq \text{const}$. Thus, generally speaking, the simple expression (2) for fully additive action of the gravitation and the magnetic buoyancy on convection and heat transfer is not valid.

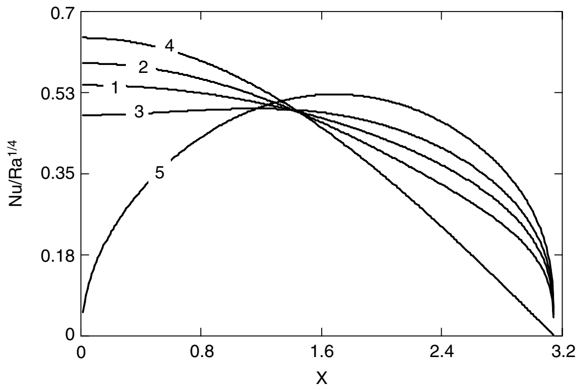

Figure 1. Distribution of the heat transfer coefficient on the cylinder surface (the coordinate $x = 0$ corresponds to the frontal critical point of convective flow) at various values of a: 1, 0; 2, +0.2; 3, −0.2; 4, +0.5; 5, −0.5.

Let us consider the heat transfer due to a combined gravitation and magnetic convection on the horizontal circular cylinder. The cylinder axis is directed along Cartesian coordinate z'. The nonuniform magnetic field is oriented transversely to the cylinder axis along the coordinate y': $H_y = H_0(x')$. The field has a constant vertical gradient $\nabla_x H_y = H_0 m$. Maxwell equation allows such geometry of magnetic fields if the scalar magnetic potential is equal to

$$\Psi = H_0 y'(1 - mx'). \quad (4)$$

Thus, besides the horizontal component of the magnetic field $H_{y'} = H_0(1 - mx')$ there appears also a vertical component $H_{x'} = -H_0 m y'$. In our experiments the cylinder is placed in the central section of a gap between magnetic poles located at $y' = \pm\delta$. In order to ensure the field gradient m, the cylinder is located near the top or bottom ends of the gap. The diameter of cylinder $d \ll \delta$; therefore the relative vertical gradient of the magnetic field m near its surface is nearly constant and the scalar magnetic potential may be considered equal to (4) with H_0 being the field in the center of the cylinder. Now, the corresponding function $\alpha(x)$ is the following:

$$\alpha(x) = \sin x - a \sin 2x \quad (5)$$

with $a = md/2$. Curves of local magnetoconvective heat transfer on the cylinder surface calculated from (3) for various values of the parameter a are plotted in figure 1 (the gradient m is assumed approximately equal to $1/\delta$). From the presented results one can see that the non-homogeneity of magnetic acceleration around the cylinder causes only relatively small changes in the local heat transfer.

Magnetic field gradients induced by bodies themselves play a more important role, even if the external field is uniform. A cylinder with magnetic permeability μ_i, different from that of a magnetic fluid μ_a, induces local field perturbations. The corresponding scalar magnetic potential now is equal to

$$\psi = H_0 \frac{K_\mu}{r} K_\mu \cos\vartheta. \quad (6)$$

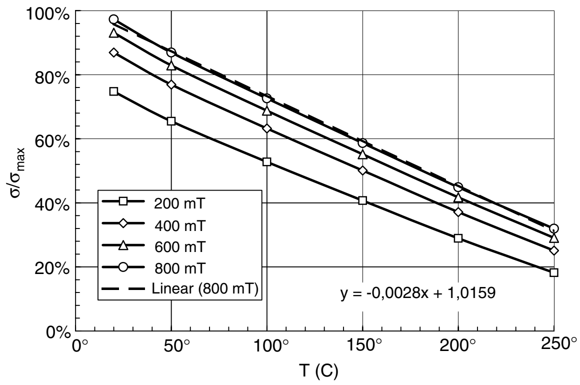

Figure 2. The fluid DF-67 relative magnetization dependence on temperature. The linear approximation performed gives the value of the pyromagnetic coefficient $\beta_m = -0.0028\ \mathrm{K}^{-1}$.

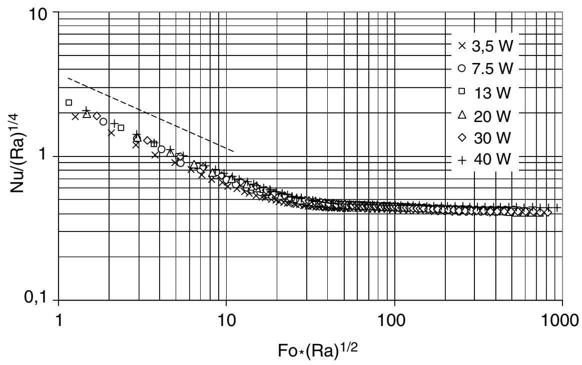

Figure 3. Stabilization of heat transfer without a magnetic field applied. The series represents various heating power values. The line represents the dependence (7) for a porcelain cylinder.

Here $K_\mu = (\mu_a - \mu_i)/(\mu_a + \mu_i)$, $r = 2R/d$ is the relative radial coordinate, the polar angle ϑ characterizes the direction of external magnetic field H_0. The function $\alpha(x)$ again is equal to (5) only with another coefficient a [6]. For nonmagnetic cylinders, $a = \pm\mu_0 M^2 \beta_m (\beta_T \rho g d)^{-1}$. Positive a values correspond to a horizontal magnetic field direction, negative a to its vertical direction. The curves presented in figure 1 predict a strong influence of $\alpha(x)$ on the local heat transfer, whereas the total heat flux changes insignificantly. The boundary layer approximation and (4) can be used only at $\alpha < 0.5$ until the magnetic pressure gradient is less than the gravitation one. Numerical estimates show that this coefficient can reach significantly higher values $a > 1$ even in the presence of relatively small fields. This means that the radial magnetic pressure gradient near the cylinder surface is very high and the boundary layer approximation is no longer valid. Around the cylinder develop convective flames symmetric about the direction of the applied field. They proliferate at great distances from the body [4]. Therefore in the case of transversely oriented magnetic field there is only one way to perform theoretical investigations of the heat transfer numerically.

The influence of local magnetic field gradients can be eliminated if the cylinder is oriented along the direction of magnetic field and the demagnetization factor is low.

3. Experimental details

The experimental setup represents the cooling of a cylindrical heater, 50 mm in length and $d = 5$ mm in diameter, which is immersed in the center of a vessel with thermostabilized (17 °C) walls, filled with a magnetic fluid. The cylinder is made of ceramic (porcelain); a thin bifilar copper wire coil heats it electrically. The temperature of the heater is determined from measurements of the electric resistance of the coil. The power of the heater is varied in the interval of approximately 4–40 W. The coil temperature T_w does not exceed 50 °C at maximal power. Separate average temperature measurements of the surrounding liquid in the vessel allow us to conclude that the power of the thermostat is sufficient to keep the average temperature of liquid constant and equal to that of the vessel walls T_0. Therefore, the Nusselt number Nu is determined by assigning the power density of the heater to the temperature difference $\Delta T = T_w - T_0$.

The experiments are carried out employing a tetradecane based temperature-sensitive ferrofluid stabilized by oleic acid (sample DF-67, Institute of Physics, LU). The magnetic phase of the colloid is complex ferrite $Fe_2Mn_{0.5}Zn_{0.5}O_4$ of room temperature magnetization 224.4 kA m^{-1}. The magnetic diameter of the particles is 11.8 nm (determined by a magnetogranulometry technique); their volumetric phase concentration $\varphi = 0.045$. The density of the colloid $\rho = 976$ kg m^{-3}, the viscosity at room temperature $\eta = 7.11 \times 10^{-3}$ N s m^{-2}, and the magnetization at $T = 20$ °C near magnetic saturation is $\sigma = 10$ kA m^{-1}. Due to the low φ, other coefficients necessary for calculating Nu, Ra_{T} and Ra_{m} numbers are taken equal to those of the liquid carried. Figure 2 represents the dependence of fluid magnetization on temperature. Manganese–zinc ferrite particles have strong pyromagnetic properties. For all magnetizing fields the relative pyromagnetic coefficient is approximately one and the same, and equal to $\beta_m = 0.0028\ \mathrm{K}^{-1}$.

4. Results and discussion

Figure 3 illustrates the results of heat transfer measurements for pure thermogravitation convection without applying the magnetic field ($Tm_{\mathrm{T}} = 0$). These measurements relate to non-stationary convection, starting from the moment the heater is switched on, and finishing after reaching the steady state. According to the theory of the free non-stationary convection [2], the curves which represent different dissipated heating powers should coincide with one another in the non-stationary stage as well as in the steady state, in the chosen coordinates ($Fo = \lambda t/(\rho c_{\mathrm{T}} d^2)$ is the thermal Fourier number). In the initial stage of unsteady convection the heat transfer does not depend on the convection velocity; the Nu number follows the dependence $Nu\sqrt{Fo} = k =$ const of a pure thermal conduction regime. The boundary layer theory for the given experiment conditions (constant heat flux) gives the value of this coefficient as equal to $\sqrt{\pi}/2$. Measurement results presented in figure 3 indicate significantly higher heat transfer intensity in the initial regime; the coefficient $k \approx 2.5$. Obviously, this reflects the specifics of the experiment. The heater is located on the surface of the cylinder; therefore in the initial regime the thermal boundary layer develops not only in the fluid but also inside the cylinder. Accumulation of heating energy inside the cylinder hinders a growth of the surface temperature. During this period the heat transfer intensity follows the dependence

$$Nu\sqrt{Fo} = \frac{\sqrt{\pi}}{2}\left(1 + \sqrt{\frac{\rho_i c_i \lambda_i}{\rho c \lambda}}\right). \tag{7}$$

Symbols with index i here denote the physical properties of the cylinder. Due to high thermal conductivity of the cylinder (porcelain), the constant k in the unsteady heat transfer equation (7) is equal to 3.84. It is higher than the experimentally obtained value 2.5. The inner thermal boundary layer reaches the center of the cylinder at $Fo_i \approx 1$. This value corresponds to a real time of about 10 s. This time is nearly equal to that for the heat transfer curve starting to saturate due to the convection. Thus, the unsteady external heat transfer curve corresponds to a partially stabilized inner temperature; therefore the real coefficient k may be less than that predicted by (7).

The steady heat transfer sets in at $Fo\sqrt{Ra_{\rm T}} \approx 40$. In accordance with theoretical predictions the curves for all heating powers coincide (a slight spreading can be explained mainly due to the dependence of the fluid viscosity and thermal conductivity on temperature, which is not taken into account in the analysis). For the heat transfer coefficient we obtain (the Prandtl number of the colloid is $Pr = 90$)

$$Nu = 0.45 Ra_{\rm T}^{0.25} \tag{8}$$

whereas the coefficient in this dependence for the total Nusselt number calculated from formula (3) is equal to 0.529. Taking into account the difference in boundary conditions (contrary to the conditions of the experiment, the dependence (3) corresponds to a constant wall temperature), one can conclude that the experimental results agree relatively well with the theoretical predictions.

The results of heat transfer measurements under the influence of a nonuniform magnetic field are presented in figure 4. The geometry of the applied field corresponds to that explained in section 2. In order to examine the additive action of thermogravitational and thermomagnetic convection on heat transfer, the measurement results are normalized by using the total Rayleigh number $Ra = Ra_{\rm T} + Ra_{\rm m}$. Two series of experiments are performed. The first one, which corresponds to parallel orientation of gravitation and magnetic forces, shows a monotonic augmentation of heat transfer intensity. The second series, performed under opposite orientation of the two forces, indicates a minimum of the heat fluxes. This minimum corresponds well to the magnetic field intensity at which the magnetic body force nearly compensates the gravitation force. Taking into account the parameters of the experiment, from (2) it follows that the balance between the gravitation and magnetic buoyancy forces should appear at $H_0 = 36.5$ kA m^{-1} which is very close to that observed in the experiment; see curve 2 in figure 4. Nevertheless, the convincing evidence of equivalence between the gravitation and the magnetic convections is not established. From the curves in figure 4 one can see that for mixed convection the power in formula (8) is less than 0.25. One reason for such discrepancy between the heat transfer laws for magnetic convection and for gravitation may be an influence of local magnetic forces near the heat transfer surface due to field perturbations induced by the nonmagnetic body and described in section 2. Numerical evaluation shows that the parameter a in (6) in the presence of strong fields in our experiment exceeds the value 10. Under such conditions the heat transfer theories based on boundary layer approximation cannot be applied. It is necessary to account for complicated structures of convective flames induced by the bodies. The influence of the self-magnetic field on the convection can be hindered if the cylinder is oriented along the direction of the applied field (parallel to coordinate y'). Really, the corresponding heat transfer measurements (see the curve 3 in figure 4) are evidence of lowering of the non-additivity in the action of $Ra_{\rm T}$ and $Ra_{\rm m}$ on the total heat transfer coefficient.

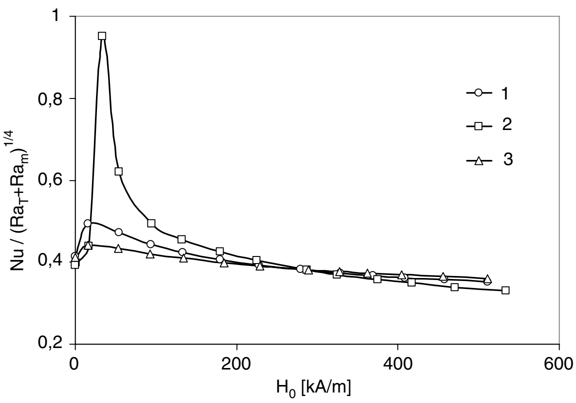

Figure 4. Dependence of the relative heat transfer intensity on the applied magnetic field strength (heating power 30 W). 1—transverse field (H_y), parallel orientation of gravitation and magnetic body forces, 2—transverse field (H_y), anti-parallel orientation of body forces, 3—longitudinal field (H_z), parallel orientation of body forces.

5. Conclusions

The experiments performed confirm the additive action of the thermogravitation and thermomagnetic body forces on the heat transfer intensity in ferrofluids. In further experiments, it is necessary to perform additional theoretical and experimental research to clarify specific convection problems related to spatial nonuniformity of the magnetic 'buoyancy' force and the appearance of local field gradients near surfaces.

Thermomagnetic convection is an effective way of achieving augmentation of heat transfer. Employing complex ferrite based ferrofluids, it is possible to reach magnetic Rayleigh numbers $Ra_{\rm m}$ which exceed the gravitation $Ra_{\rm T}$ numbers by up to 20 times. Special design and increase of the magnetic field gradients by implementation of modern permanent magnets allows reaching much higher Ra_{m} numbers and promises the augmentation of the heat transfer by 2–3 times.

Acknowledgments

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